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"Level curvature" distribution for diffusive Aharonov-Bohm systems: Analytical results

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We calculate analytically the distributions of "level curvatures" (the second derivatives of eigenvalues with respect to a magnetic flux) for a particle moving in a white-noise random potential. We find that the Zakrzewski-Delande conjecture [J. Zakrzewski and D. Delande, Phys. Rev. E **47**, 1650 (1993)] is still valid even if the lowest weak localization corrections are taken into account. The ratio of mean level curvature modulus to mean dissipative conductance is proved to be universal and equal to 2π in agreement with available numerical data.

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Nowadays it is considered to be a well established fact that spectral statistics of generic chaotic or disordered systems is adequately described by that typical for eigenvalues of large Gaussian random matrices (RM's) [1]. The range of applicability of the "Gaussian universality" to real disordered systems was considered in the pioneering papers by Efetov [2] and Altshuler and Shklovski [3]. It was demonstrated that statistical properties of the energy levels of a quantum particle moving in a static random potential follow RM predictions as long as effects of Anderson localization on the particle diffusion are negligible. Another important result is due to Berry, who demonstrated how the same universality may arise in globally chaotic ballistic systems ("quantum billiards") [4].

Quite recently, an interesting development in the study of spectra of disordered systems and their chaotic counterparts has been made. The problem is to study the so-called "level response statistics," i.e., to provide a statistical description of sensitivity of the energy levels to external perturbations of different types. This issue attracted a great deal of research interest, both analytically and numerically [5–9,11–20]. The most frequently studied characteristics are first and second derivatives of the energy levels $E_n(\alpha)$ with respect to a tunable parameter α characterizing the strength of perturbation. Physically the role of such a parameter can be played by, e.g., an external magnetic field, the strength of a scattering potential for disordered metal, a form of confining potential for quantum billiards, or any other appropriate parameter on which the system Hamiltonian is dependent. The first derivatives $v_n=\partial E_n/\partial\alpha$ are frequently called the "level velocities" (LV's) (or "level currents"), the second ones $K_n=\partial^2 E_n/\partial\alpha^2$ are known as "level curvatures."

In a series of papers by Altshuler and co-workers [6,8], see also [7], it was found that for a generic chaotic system whose unperturbed spectrum is well described by the universal RM statistics the set of level velocities $v_n(\alpha)$ is characterized (after appropriate rescaling) by a universal correlation function $\langle v_n(\alpha)v_n(\alpha')\rangle$ whose form is again dependent only on the symmetry of the unperturbed Hamiltonian and that of the perturbation.

The range of applicability of these results to real systems is the same as before: they are valid for systems with completely "ergodic" chaotic eigenfunctions covering randomly, but uniformly all the available phase space and showing no specific internal structure. The universality is believed to be independent of "whether the chaos originates in mesoscopic disorder or deterministic instability of the classical trajectories" [9]. The effects of eigenfunction localization—either due to scarring [10] or due to disorder-induced quantum interference (the Anderson localization)—result in substantial modifications of the LV characteristics; see [11–13,15].

Much interest was concentrated on the level curvature (LC) distribution $\mathcal{P}(K)$. Gaspard and co-workers [14] discovered the universal asymptotic behavior $\mathcal{P}(K) \propto K^{-(\beta+2)}$ in the large-curvature limit $K\to\infty$, the parameter $\beta=1$, 2, or 4 depending on the symmetry universality class. This behavior is a direct consequence of the so-called "level repulsion" for unperturbed systems. Namely, the probability $\mathcal{P}(s)$ for two unperturbed energy levels to be separated by the spacing s much smaller than the mean level spacing Δ vanishes as $\mathcal{P}(s)\propto s^\beta$, and this fact can be shown to result in the above-mentioned large-curvature behavior; see, e.g., [14,16]. The analytical form of the whole distribution function $\mathcal{P}(K)$ was guessed by Zakrzewski and Delande [11] on the basis of numerical results and is given by the following expression [17]:

$$\mathcal{P}(k)=C(\beta)(1+k^2)^{-(1+\beta/2)}, \tag{1}$$

where C_β is a normalization constant, $\beta=1,2,4$ depending on the presence or absense of time-reversal invariance and Kramers degeneracy, and k is the dimensionless level curvature. We will refer to this expression as the ZD conjecture.

Very recently, von Oppen [18] succeeded in demonstrating that Eq. (1) is indeed exact for the ensemble of large Gaussian Hermitian matrices ($\beta=2$) and announced results for two other symmetry classes. The validity of the ZD conjecture for all three classes of large Gaussian matrices was proved in a different way by the present authors [28]. It is natural to suppose that the status of the distribution Eq. (1) within the theory of disordered and chaotic systems is the same as before: it is valid for systems with completely ergodic extended eigenstates. This was indeed found to be the case in a series of interesting numerical experiments [19,20] on quasi-one-dimensional as well as three-dimensional (3D) periodic random tight-binding models subject to the influ-

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ence of Aharonov-Bohm magnetic flux, which acts as a time-reversal symmetry breaking parameter. In particular, the distribution Eq. (1) was found to persist up to the Anderson localization transition [19].

Unfortunately, the methods used in [18,28] for deriving the form of LC distributions for the Gaussian ensembles were heavily based upon the explicit form of the joint probability density of eigenvalues known for all these ensembles [1], which is of course an immense simplification. One therefore has to invent a different technique in order to be able to treat the curvature distribution analytically under more realistic assumptions.

It turns out to be possible to find such a technique for the case of time-reversal invariant systems subject to a time-reversal symmetry breaking perturbation. This case seems to be one of the most interesting from the physical point of view as well as relevant experimentally. As a physical realization one can imagine a disordered mesoscopic sample (e.g., cylinder or ring) pierced by magnetic flux ϕ. For such a system the "typical" level curvature is expected to be related to the dimensionless conductance g_c of the sample due to the famous Thouless formula: $(1/\Delta)\partial^2E_n/\partial\phi^2|_{\phi=0}\sim E_c/\Delta\equiv g_c$. Initially suggested by Thouless [21], this relation attracted renewed interest recently. Its meaning was reconsidered in a broader context by Akkermans and Montambaux [22]; see also a quite detailed discussion in [16,20]. All these facts make the consideration of such systems to be of special interest.

A specific feature allowing us to treat the particular case of weak time-reversal symmetry breaking perturbation analytically is the vanishing of the first derivatives $\partial E_n/\partial\phi|_{\phi=0}$ for reasons of symmetry. This fact allows one to represent the curvature distribution in terms of a product of advanced and retarded Green functions, the average over the disorder being performed nonperturbatively with the help of Efetov's supersymmetry approach [2]. The method provides a unique possibility to derive the level curvature distribution starting from a genuine microscopic Hamiltonian of a quantum particle experiencing elastic scattering:

$$\mathscr{H}(\phi)=(1/2m)[\mathbf{p}-(e/c)\mathbf{A}(\phi)]^2+U(\mathbf{r}), \tag{2}$$

with $U(\mathbf{r})$ being a white-noise random potential and $\mathbf{A}(\phi)$ standing for the vector potential corresponding to the magnetic flux ϕ.

This fact allows one to try to take into account the weak localization effects due to the finite ratio of the Thouless energy E_c to the mean level spacing Δ. The general way of doing this was developed recently by Kravtsov and Mirlin [24]. Exploiting this method one can find that the curvature distribution Eq. (1) preserves its form to the first order in $\Delta/E_c\ll 1$, the width $\langle|K|\rangle$ being renormalized.

To begin with, we introduce the resolvent operator $\hat{G}^{\pm}(\alpha;\epsilon)=[E-\mathscr{H}(\phi)\pm i\epsilon]^{-1}$ where $\alpha=\phi/\phi_0$, with $\phi_0=2\pi c/e$ being the flux quanta. Let us now consider the following correlation function:

$$\mathscr{K}(u)=\lim_{\epsilon\to 0}\epsilon\mathrm{Tr}\hat{G}^{+}(\alpha=2\sqrt{\epsilon/u};\epsilon)\mathrm{Tr}\hat{G}^{-}(\alpha=0;\epsilon)$$

$$\equiv\lim_{\epsilon\to 0}\sum_{n,m=1}^{N}\frac{\epsilon}{[E-E_n(\alpha=0)-i\epsilon][E-E_m(\alpha=2\sqrt{\epsilon/u})+i\epsilon]}. \tag{3}$$

Here we used the expression for the trace of the resolvent $\mathrm{Tr}\hat{G}$ in terms of eigenvalues of the Hamiltonian $\hat{H}(\phi)$.

It is important to note that the limiting procedure $\epsilon\to 0$ in Eq. (3) is performed prior to the thermodynamic limit $V\to\infty$, where V is the system volume. Therefore the parameter ϵ (that plays a role of effective "level broadening" necessary to regularize the resolvent operator) can be considered as small in comparison with the mean level spacing $\Delta\propto 1/V$. It was already mentioned that the probability of having two levels at a distance $s\ll\Delta$ is vanishingly small due to level repulsion. Taking this fact into account, one finds that the only terms that survive the limiting procedure $\epsilon\to 0$ are those with coinciding indices $m=n$. Remembering $\partial E_n/\partial\phi|_{\phi=0}$, one obtains

$$\mathscr{K}(u)=\pi\sum_n\frac{u^2+iuK_n}{u^2+K_n^2}\,\delta(E-E_n),\quad K_n=\frac{\partial^2E_n}{\partial\alpha^2}\bigg|_{\alpha=0}. \tag{4}$$

Let us perform formally the averaging over the disorder and introduce the function $\mathscr{P}(K)=\langle\Delta\sum_{n=1}^{N}\delta(K-K_n)\delta(E-E_n)\rangle$. This function has the meaning of distribution of curvatures for levels in a narrow spectral window around the energy E . Then one obtains the following relation:

$$\frac{\Delta}{\pi}\langle\mathscr{K}(u)\rangle=u\int_0^\infty dve^{-uv}\int_{-\infty}^{\infty}dK\mathscr{P}(K)\exp(ivK). \tag{5}$$

Let us stress that it is simultaneous vanishing of all the first derivatives $v_n=\partial E_n/\partial\phi$ at $\phi=0$ that made it possible to scale $\phi/\phi_0\propto\epsilon^{1/2}$ producing the finite limiting expression Eq. (4) when $\epsilon\to 0$ in Eq. (3). In contrast, when $v_n\neq 0$ one can scale $\phi\propto\epsilon$ and get the expression Eq. (5), but with the *level velocity* distribution $\mathscr{P}(v)$ substituted for the *level curvature* distribution $\mathscr{P}(K)$. This fact was already used in [12] (see closely related method in [8,23]) in order to calculate analytically LV distributions for different systems.

In order to evaluate the average correlation function $\mathscr{K}(u)$ one can use Efetov's supersymmetry approach [2]. Exploiting the weak disorder parameter $k_Fl\gg 1$, with k_F denoting the Fermi wave number and l denoting the mean free path due to elastic scattering, the problem can be mapped onto the so-called nonlinear graded σ model. Rather detailed exposition of the mapping can be found, e.g., in [26]. The final expressions appeared frequently in the literature, most recently in [25], and can be used for our needs after a slight modification resulting from the fact that in our case the magnetic flux enters only one of two Green functions rather than both as in [26,25]. As the result, one finds

$$\langle\mathscr{K}(u)\rangle=\frac{\pi^2}{4}\lim_{\epsilon\to 0}\frac{\epsilon}{\Delta^2}\int\mathscr{D}\mu(Q)\int\frac{d\mathbf{r}}{V}\mathrm{Tr}_g\left[\hat{Q}(\mathbf{r})\hat{k}\,\frac{1+\Lambda}{2}\right]\times\int\frac{d\mathbf{r}}{V}\mathrm{Tr}_g\left[\hat{Q}(\mathbf{r})\hat{k}\,\frac{1-\Lambda}{2}\right]\exp-\mathscr{S}_\epsilon(Q), \tag{6}$$

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$$\mathcal{S}_\epsilon(Q) = -\frac{\pi D}{8\Delta}\int \frac{d\boldsymbol{r}}{V}\mathrm{Tr}_g\left(\nabla Q - \frac{e}{c}\boldsymbol{A}_\epsilon[\hat{Q},\hat{\tau}]\right)^2 - \frac{\pi\epsilon}{2\Delta}\int \frac{d\boldsymbol{r}}{V}\mathrm{Tr}_g\hat{Q}\hat{\Lambda},$$

where D is the classical diffusion constant $D \propto v_F l$ due to random scattering , $\boldsymbol{A}_\epsilon$ is the vector potential corresponding to the magnetic flux through the sample $\phi/\phi_0 = 2\sqrt{\epsilon/u}$ in the gauge ensuring that $\boldsymbol{A}$ is tangential to the surface of the sample; e.g., $(e/c)\boldsymbol{A} = (2\pi/L)(\phi/\phi_0)\boldsymbol{e}_\tau$ for the ring geometry. The notation [,] stands for the matrix commutator and Tr_g stands for the graded trace. The position dependent 8×8 supermatrices $\hat{Q}(\boldsymbol{r})$ satisfy the constraint $\mathrm{Tr}_g\hat{Q}^2 = 1$ and can be parametrized as follows: $\hat{Q} = \hat{T}^{-1}\Lambda\hat{T}$, where $\hat{T}$ belongs to a graded coset space UOSP(2,2/4)/UOSP(2/2)×UOSP(2/,2). Other 8×8 supermatrices entering the Eq. (6) are diagonal:

$$\hat{\Lambda} = \mathrm{diag}(I_2, I_2, -I_2, -I_2), \quad \hat{\tau} = \mathrm{diag}(\hat{\sigma}, \hat{\sigma}, 0,0,0,0),$$

$$\hat{k} = \mathrm{diag}(I_2, -I_2, I_2, -I_2), \quad \hat{\sigma} = \mathrm{diag}(1,-1), \quad I_2 = \mathrm{diag}(1,1).$$

Let us now recall that there are two relevant energy scales for a quantum particle diffusing in a closed disordered sample of the size $L > l$: the mean level spacing Δ and the Thouless energy $E_c = D/L^2$. For $\Delta \ll E_c$ it is known [2] that the main contribution to the functional integral, Eq. (6), comes from spatially uniform configurations (a so-called "zero mode"): $\hat{Q}(\boldsymbol{r}) = \hat{Q}_0 \equiv \hat{T}_0^{-1}\hat{\Lambda}\hat{T}_0$. The condition $g_c = E_c/\Delta \gg 1$ determines the regime where statistical characteristics both of eigenfunctions and energy levels of the Hamiltonian Eq. (2) are adequately described by classical RM theory [2,3,13]. The corrections to RM results have a form of regular expansion in the small parameter g_c^{-1}. A systematic way to construct such an expansion was originally suggested in [24] and discussed in more detail in [13]. The general procedure is as follows: one decomposes the matrix $\hat{Q}(\boldsymbol{r})$ as $\hat{Q}(\boldsymbol{r}) = \hat{T}_0^{-1}\tilde{Q}(\boldsymbol{r})\hat{T}_0$ and uses the fact that supermatrices $\tilde{Q}(\boldsymbol{r})$ only weakly fluctuate around the value $\tilde{Q} = \hat{\Lambda}$ as long as $g_c \gg 1$. As a result, the integration over $\tilde{Q}(\boldsymbol{r})$ in Eq. (6) can be performed perturbatively. After that, one obtains a renormalized expression for the "zero mode effective action" $S_\epsilon(\hat{Q}_0)$ governing the integral over spatially uniform configuration $\hat{Q}_0$. This last integration has to be performed nonperturbatively.

Performing the "perturbative" part of the evaluation of the functional integral to the first order in the small parameter g_c^{-1} and keeping only terms relevant at $\epsilon \to 0$ one obtains for the quantity $\langle \mathcal{K}(u)\rangle$ the same expression Eq. (6) with the following replacement: $\hat{Q}(\boldsymbol{r}) \to \hat{Q}_0$, $S_\epsilon(\hat{Q}) \to S_\epsilon(\hat{Q}_0)$, where

$$S_\epsilon(\hat{Q}_0) = -\frac{\pi\epsilon}{2\Delta}\mathrm{Tr}_g(\hat{Q}_0\hat{\Lambda}) - \frac{\pi\epsilon}{u\Delta}bE_c\left(1 - \int \frac{d\boldsymbol{r}}{V}\Pi(\boldsymbol{r},\boldsymbol{r})\right)\mathrm{Tr}_g(\hat{Q}_0\hat{\tau}\hat{Q}_0\hat{\tau}). \tag{7}$$

Here we introduced a sample-dependent geometrical factor $b = (u/4\epsilon)L^2\int(d\boldsymbol{r}/V)[(e/c)\boldsymbol{A}_\epsilon]^2$, see [25], equal for the ring geometry to $b = 4\pi^2$. In the expression Eq. (7) the quantity $\Pi(\boldsymbol{r},\boldsymbol{r}')$ stands for the so-called diffusion propagator whose Fourier transform is equal to $(1/\pi\nu)[1/(D\boldsymbol{q}^2 + \epsilon)]$, momentum $\boldsymbol{q}$ going over all *nonzero* values allowed from the quantization condition in a given sample geometry, ν being the mean density of states at Fermi level.

In order to evaluate the integral over $\hat{Q}_0$ nonperturbatively one has to use the so-called Efetov parametrization of the corresponding coset space. A useful discussion of necessary technical details can be found, in particular, in the paper [27]. Below we present only major steps of the evaluation procedure, relegating most of the details to a more extended publication [28].

After standard manipulations, the integration over the graded coset space is reduced to performing the following threefold integral:

$$\frac{\Delta}{\pi^2}\langle \mathcal{K}(u)\rangle = \lim_{\tilde{\epsilon}\to 0}\tilde{\epsilon}\int_{-1}^{1}d\lambda\int_1^\infty\int_1^\infty d\lambda_1 d\lambda_2 \frac{(1-\lambda^2)(\lambda-\lambda_1\lambda_2)^2}{[\lambda_1^2+\lambda_2^2+\lambda^2-2\lambda\lambda_1\lambda_2-1]^2}\left[1 - \frac{2\tilde{\epsilon}}{\tilde{u}}(\lambda_1^2+\lambda_2^2+\lambda^2-2\lambda\lambda_1\lambda_2-1)\right]$$
$$\times\exp\left[-2\pi\tilde{\epsilon}(\lambda_1\lambda_2-\lambda) - \pi^2\frac{\tilde{\epsilon}}{\tilde{u}}(\lambda_1^2+\lambda_2^2-\lambda^2-1)\right] \tag{8}$$

where we introduced the notations

$$\tilde{u} = u\left(\frac{2bE_c}{\pi}\left[1 - \int \frac{d\boldsymbol{r}}{V}\Pi(\boldsymbol{r},\boldsymbol{r})\right]\right)^{-1}, \quad \tilde{\epsilon} = \frac{\epsilon}{\Delta}.$$

A close inspection of this expression makes it clear that the only contribution nonvanishing in the limit $\tilde{\epsilon}\to 0$ is that coming from the region $\lambda_1 \sim \lambda_2 \propto \tilde{\epsilon}^{-1/2}$. Picking up such a contribution one gets, after some algebra,

$$\frac{\Delta}{\pi}\langle \mathcal{K}(u)\rangle = \frac{\tilde{u}}{2\pi}\int_0^\infty dv\, v\, e^{-(2\tilde{u}/\pi)v}\int_0^\infty \frac{dy}{y}e^{-v(y+y^{-1})} \times \int_{-1}^{1}d\lambda \frac{\left[2(1-\lambda^2) + \frac{\lambda}{v}\right]}{(y+y^{-1}-2\lambda)}. \tag{9}$$

The two last integrations in this expression can be performed analytically. Comparing the result with the relation Eq. (5) one obtains the following expression for the Fourier transform of the LC distribution:

$$\int_{-\infty}^{\infty} \mathscr{P}(K)\exp(ivK)dK=\frac{\gamma}{2}\,vK_1(\gamma v), \quad (10)$$

$$\gamma=2bE_c\left(1-\int\frac{dr}{V}\,\Pi(\boldsymbol{r},\boldsymbol{r})\right), \quad (11)$$

where $K_1(z)$ is the Macdonald function. Inverting this Fourier transform one finally arrives at the desired level curvature distribution:

$$\mathscr{P}(K)=\frac{1}{2}\,\frac{\gamma^2}{[K^2+\gamma^2]^{3/2}}\,. \quad (12)$$

Thus we proved that the level curvature distribution for weakly disordered Aharonov-Bohm systems follows *exactly* the form suggested by Zakrzewski and Delande [11] even beyond the domain of validity of random matrix theory, i.e., when the lowest weak localization corrections are taken into account. This conclusion is in good coordination with available data from recent papers [19,20] where numerical simulations of lattice analogs of disordered Aharonov-Bohm systems were performed. Having in mind a connection between asymptotic behavior of LC distribution and degree of level repulsion, it is worth mentioning that our result is also in agreement with the recent paper [24], where the two-level correlator was shown to be affected by weak localization corrections only at the second order with respect to g_c^{-1}.

Another interesting point is that our exact results when combined with the earlier findings [6,8] allow one to prove the universality of the ratio of the LC mean scaled modulus $\langle|K|\rangle/\Delta=\gamma/\Delta$ to the sample dissipative conductance [6,20,22] defined as $C(0)=(1/\Delta^2)\langle(\partial E_n/\partial\alpha)^2\rangle$, where the average goes both over the flux value and over the disorder. The calculation done in [6] produces the value $C(0)=bE_c/(\pi\Delta)$ for any Aharonov-Bohm diffusive system when $g_c\gg 1$. Comparing this result with Eq. (11) one finds $\gamma/\Delta C(0)|_{g_c\gg 1}=2\pi$ in good agreement with the value $6.7\pm 10\%$ found in numerical simulations [20].

Finally, let us mention that our method allows one to extend the validity of the ZD conjecture to large symmetric RM's with independent *arbitrarily* distributed entries having finite variance. In particular, the most interesting ensemble of large *sparse* RM's [29] can be treated analytically in this way. These results will be published elsewhere [28].

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